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TURBULENT FREE CONVECTION FLOW

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Abstract—The results of an experimental investigation of a turbulent free convection boundary layer are presented for a fluid with a low Prandtl number (mercury). The boundary layer was formed along the isothermally heated vertical wall of a cell.

The measured mean temperature profiles and turbulent temperature distributions, indicated the existence of a fully developed boundary layer.

In the recorded temperature spectra the high frequencies region, associated with dissipation, was found to exhibit an f^{-3} dependence (f is the frequency of the temperature fluctuations). The presence and development of a "convection subrange" was also identified, dependent upon the Rayleigh number and the location of the point of measurement with respect to the wall.

NOMENCLATURE

c_p, specific heat;
d, width of the cell;
g, acceleration of gravity;
Gr_x, local Grashof number, $= \dfrac{g\beta\Delta\theta_w x^3}{\nu^2}$;
Gr_d, Grashof number based on the width of the cell, $= \dfrac{g\beta\Delta\theta_c d^3}{\nu^2}$;
H, heat flux;
k, wavenumber;
L, length of the heated wall of the cell;
Pr, Prandtl number, ν/α;
Q, constant flux of buoyancy, $= H/c_p\theta$ (see also [32]);
Ra_x, local Rayleigh number, $= Gr_x Pr$;
Ra_d, Rayleigh number based on the width of the cell, $= Gr_d Pr$;
U_i, mean velocity components;
u_i', turbulent velocity components;
u_0, velocity scale, $= (\alpha g Q)^{\frac{1}{4}}$ (see also [32]);
x, y, coordinates along and perpendicular to the heated wall;
$y_{0.5}$, distance from the wall at which turbulent temperature intensity dropped to half of its maximum value;
z_0, length scale, $=(\alpha^3/gQ)^{\frac{1}{4}}$ (see also [32]);
α, thermal diffusivity;
δ, thickness of the thermal boundary layer;
ϵ, turbulent dissipation;
θ, mean temperature;
θ', temperature fluctuations;
θ_w, temperature at the wall;
θ_∞, temperature outside the boundary layer;
θ_0, temperature scale, $= (Q^3/\alpha g)^{\frac{1}{4}}$ (see also [32]);
$\Delta\theta_w$, temperature drop across the thermal layer, $= \theta_w - \theta_\infty$;
$\Delta\theta_c$, temperature drop across the width of the cell (taken approximately equal to $2\Delta\theta_w$);
Θ, non-dimensional parameter defined as $\dfrac{\theta - \theta_\infty}{\theta_w - \theta_\infty}$;
ν, kinematic viscosity;
ρ, density.

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INTRODUCTION

TURBULENT free convection is a rather common phenomenon in nature. Its importance arises from its frequent occurrence not only in technological problems but also in problems related to Geophysical Sciences.

The subject of the present experiment was a free convection turbulent boundary layer formed along an isothermally heated vertical wall of a stainless steel cell filled with mercury.

The measured quantities were mean temperature profiles, temperature turbulent intensity distributions and temperature turbulent spectra. These are quantities characterizing the thermal boundary layer. Because a buoyant flow involves an interdependence of the velocity and temperature fields, it was hoped that the study of the thermal boundary layer would also reveal some aspects of the nature of the velocity field.

The present study merits importance because of lack

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of available measurements of any statistical quantities which would permit the understanding of the structure of the turbulent field in a free convection flow. Also, to the best of the authors knowledge, no experimental measurements, except for some overall heat transfer rates, have been reported in the literature on turbulent free convection flow for a fluid with low Prandtl number.

Notwithstanding the fact that in the present work a specific geometry is considered, it is hoped that the results obtained would be helpful in the interpretation of more general situations.

LITERATURE REVIEW

1. The majority of the theoretical attempts, aiming at obtaining a solution for the free convection turbulent flow along a vertical isothermal flat plate, are based on the momentum integral method. Theoretical works of this kind are those of Eckert and Jackson [1] where the experimental results of Griffiths and Davis [2] for air were used, Bayley [3] in which fluids of low Prandtl numbers were also considered and Fujii [4]. Recently, Yank and Nee [5] attempted to solve the problem by introducing the ideas of Nee and Kovaszney [6] concerning the change of total viscosity (molecular plus eddy) in the boundary layer. Their results pertain mostly to the structure of the free convention turbulent flow.

The lack of an adequate understanding of the problem must be attributed primarily to the fact that the existing experimental data are limited. They consist mainly of overall heat transfer rate measurements and mean velocity and temperature profiles [7–9]. Only recently, measurements of temperature fluctuations and intermittency were reported by Lock and Trotter [10]. Their conclusions will be discussed later in this work.

2. The existing analytical works on the turbulent free convection flow in a cell are those of Bachelor [11] and Emery [12]. Bachelor, described the parameters which uniquely determine the flow in the cell to be the Prandtl number, Rayleigh number (based on the width of the cell) and the ratio of the sides of the cell, L/d. Emery found the Nusselt number proportional to the (Grashof number)$_d$ to the $\frac{2}{5}$ power.

Jacob [13] examined the experimental results of Mull and Reiher [14] obtained in a cell with air and suggested empirical relations for the laminar and turbulent regimes according to which the Nusselt number is proportional to the (Grashof number)$_d$ to the $\frac{1}{4}$ and $\frac{1}{3}$ powers respectively.

The experimental results with water and mercury presented in [12] and those with air [14], are too limited to permit any conclusion concerning the relations given in [12] and [13] for turbulent free convection flow in a cell. It was evident though that transition from laminar to turbulent flow occurs in the cell at values of (Rayleigh number)$_d$ in the range between 10^5 and 10^6. Also, both theoretically and experimentally* it was suggested that for high Rayleigh numbers, a two boundary layer flow pattern exists in the cell. The estimated values of (Rayleigh number)$_d$ attained in the present experiment were above 10^6.

3. The theoretical works of Obukhoff [17], Yaglom [18] and Corrsin [19] appear to be the first on turbulent scalar (temperature) field. They considered isotropic temperature fields interacting with also isotropic turbulent velocity field while temperature variations in the flow were assumed small so that buoyancy effects can be neglected. The Kolmogoroff's hypothesis of a universal equilibrium range in the velocity spectrum was assumed, by the above authors, to apply in the case of a turbulent scalar field. At the low wavenumber end of the equilibrium range a "convection subrange" characterized by a $-\frac{5}{3}$ dependence was also predicted, corresponding to the "inertial subrange" of the velocity spectrum.

For a fluid with $\nu \gg \alpha$ (large Prandtl numbers), Batchelor [20] predicted that the form of the scalar spectrum in the range $(\epsilon/\nu^3)^{\frac{1}{4}} \ll k \ll (\epsilon/\nu\alpha^2)^{\frac{1}{4}}$ is proportional to k^{-1}. For fluids with $\nu \ll \alpha$ (low Prandtl numbers), Batchelor, Howells and Townsend [21] found a $k^{-17/3}$ dependence of the scalar spectrum in the range $(\epsilon/\alpha^3)^{\frac{1}{4}} \ll k \ll (\epsilon/\nu^3)^{\frac{1}{4}}$. Gibson [22, 23] questioned the above discussed results for highly diffusive fluids and suggested a k^{-3} dependence for the scalar spectrum instead of the $k^{-17/3}$ proposed in [21].

The experimental data on scalar turbulent fields verify the existence of a k^{-1} region at high wavenumbers in the spectra of fluids with large Prandtl numbers [24–26]. The only experiment available in which scalar turbulent spectra of fluids with low Prandtl numbers are measured is that by Rust and Sesonske [27] for the case of mercury flow in a pipe. Their results indicate the existence of a spectral region of the form k^{-3} at high wavenumbers. A detailed discussion on the subject is presented in [28].

THE EXPERIMENT

The experimental apparatus for the study of the free convection turbulent boundary layer is shown in Fig. 1. One of the vertical walls of the cell was heated

* See the works of Carlson [15] and Eckert and Carlson [16] in which interferometry was used to investigate natural convection in air enclosed in a cell.

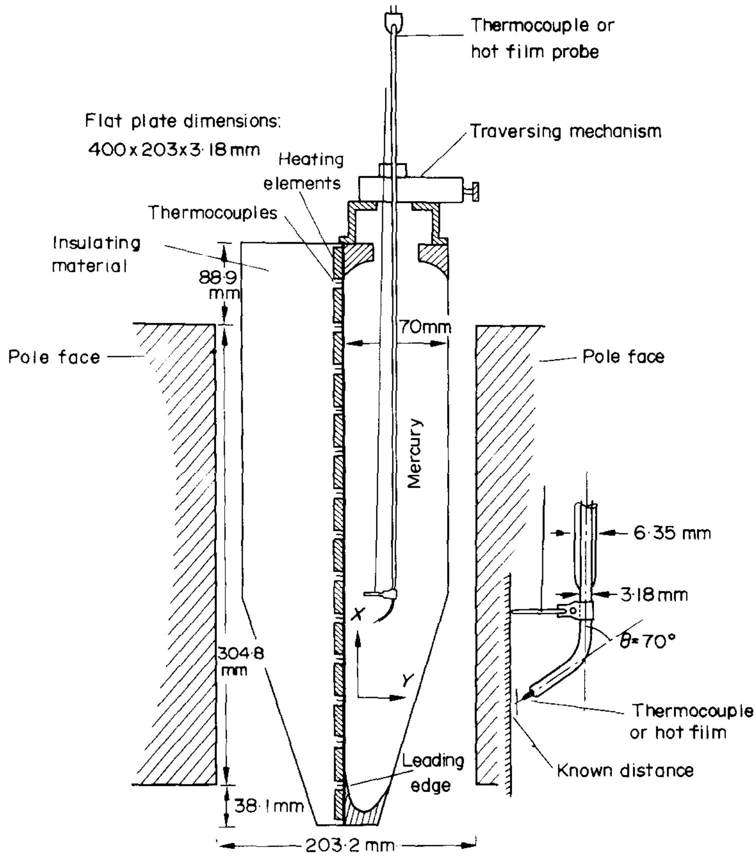

FIG. 1. General experimental set-up.

at a uniform temperature while its opposite side was cooled with water, circulating in a copper jacket attached to it. Eighteen thermocouples, distributed on the back of the heated plate were used to check the temperature distribution on it.

The cell was thermally insulated on all sides except the cooled one and it was placed between the vertically positioned pole faces of the rotating electromagnet of the Magneto-Fluid-Mechanics Laboratory.*

Two kinds of detecting sensors were employed in this experiment namely, iron–constantan thermocouples and hot film probes, for measuring temperature and temperature fluctuations respectively. The wires as well as the heads of the thermocouples were enameled with a thin coat of insulating material. The hot film sensors, products of the Thermo-Systems Inc., were coated with a thin layer of quartz. The hot film probe, operating as a resistance thermometer, was placed in the flow with the axis of the cylindrical sensor parallel to the wall. In order to detect possible errors in the measurements due to the distortion in the flow caused by the probes especially near the wall, thermocouples of diameter 0·06 and 0·08 mm were used, as well as hot film sensors of diameter 0·05 and 0·15 mm.

Depending on the kind of measurements to be made, either the thermocouple probe or the hot film were attached to a traversing mechanism which moved the probe in a direction normal to the wall. The probe could also move in the vertical direction so that the sensor could be positioned at different heights along the center line of the heated wall. The traversing mechanism was placed on the top of the cell as shown in Fig. 1. The location of the thermocouple or the hot film sensor with respect to the wall was determined by using the mechanical system shown in Fig. 1 and explained in detail in [28] and [29].

Mean temperature distributions were measured by using a Vidar integrating digital voltmeter. After several trials it was found that an integration time of

* This work is part of an investigation concerning the effect of a magnetic field on the structure of a free convection turbulent flow of a conducting fluid. The results obtained in the presence of the magnetic field will be reported in a separate communication.

200 s was sufficient to give repeatable measurements. The hot film sensor was connected to a Thermo-Systems Inc., Model 1050 hot film anemometer, which is also capable of measuring temperature. Turbulent temperature intensities were measured with a Thermo-Systems Inc. Model 1060 RMS voltmeter while for spectral measurements the signal was recorded by a Honeywell Model 7600 tape recorder. Subsequently, the spectral analysis of the recorded signal was done with the use of a Singer Model TH-26 Panoramic Subsonic analyser with an ec-36/2 recorder and wave generator and a Model POA-1 power spectral density analyser.

Power levels of 1100 and 1300 watts were applied to the cell. No attempt was made to measure the amount of heat loss. However, from laminar free convection experiments conducted in the same apparatus [29], it was found from direct estimation based on the measured temperature profiles, that the amount of heat convected by the mercury was only a few per cent (2–5 per cent) less than that applied to the heated plate. In the present experiment the heat losses are expected to be even lower than in the laminar case, due to the added cooling jacket which kept the temperature in the cell at much lower level. Generally, measurements were conducted at three stations along the wall at distances 100, 150 and 230 mm from the leading edge indicated in Fig. 1.

The wall temperature was estimated from its distribution at the back of the plate after subtracting the calculated temperature drop across it.

The presence of the probe close to the wall distorts both the velocity and temperature fields. This distortion, caused by the acceleration of the flow occurring in the space between the probe and the wall, produces a local increase in the heat convection rates with the result of measuring lower temperatures in this region. This effect has been observed in both the laminar [29] and the presence experiment in mercury and it is also discussed in [8]. Measurements of mean temperature were conducted at distances 0·10, 0·63, 1·27, 1·91 and 2·54 mm from the wall. From the last point (y = 2·54 mm), the measurements continued at intervals of 1·27 mm, until the entire thermal layer was covered. Mean temperature values at the point closest to the wall (y = 0·10 mm) were consistently found too low to fit in the measured temperature distributions. As already mentioned, this behavior is due to the proximity of the wall. Therefore, mean temperatures measured at distances closer than 0·63 mm from the wall, corresponding to about 6 per cent of the thermal layer thickness (δ = 10 mm) were not considered. This prohibited the direct estimation of local heat transfer rates at the wall.

The use of two different sizes of hot film sensors for checking the validity of the measurements near the wall showed no appreciable differences in the results.

As the intensity measurements showed a certain level of turbulence existed in the bulk of the fluid outside the thermal layer at all heights in the cell. This made difficult the definition of the thickness of the intensity distributions and consequently their non-dimensionalization based on it. The difficulty was resolved by introducing as a characteristic length the distance at which the intensity dropped to half of its maximum value.

Finally, to obtain the temperature spectra the signal representing temperature fluctuations was amplified ten times in the anemometer and fed to the recorder. Preliminary spectral analysis of the recorded temperature fluctuations showed that the upper limit of the frequencies contributing to the spectrum did not exceed the 40 c/s. Therefore, frequencies beyond 200 c/s were filtered out reducing substantially the noise level. This resulted in improving the quality of the temperature intensity measurements.

To evaluate the frequency response of the hot film sensors, the larger probe of dimater 0·50 mm (sensor with lower frequency response as compared with the one of diameter 0·015 mm) was tested in mercury. Its response to a step-like temperature change corresponded to a frequency higher than 100 c/s which is well above the upper limit of frequencies detected in the thermal layer (40 c/s).

The lowest frequency limit for the anemometer was 0·1 cycle/s. Because the lowest frequency limit for the analyser was 0·5 cyles/s the temperature fluctuations were recorded at a speed of $3\frac{3}{4}$ in/s while the signal was played back to the analyser at a speed of 30 in/s. In this way, the lowest frequency recorded (0·1 c/s) increased eight times, thus falling within the capabilities of the spectrum analyser.

A check on the spectral measurements was performed by comparing the spectral areas, representing the quantity θ'^2, with the square of the corresponding directly measured intensity. Since no absolute values of the measured spectra were obtained, only ratios of spectral areas and corresponding ratios of direct intensity readings could be compared. The values of $\overline{\theta'^2}$, obtained from the spectral areas and the intensity measurements were in good agreement, with differences of 2–6 per cent.

RESULTS AND CONCLUSIONS

The flow pattern in the cell depends on the geometry and the applied power [11]. The two regions of opposite flow direction, formed along the heated and cooled walls of the cell, can either be mixed together or form two separate layers. In the first case the flow resembles that in a two dimensional channel, while in the second case its structure is similar to that of a free convection boundary layer formed along a heated or cooled vertical plate. However, it should be noted, that there are the following differences between the flow along a vertical heated plate and that along a heated vertical wall forming a part of a cell. In the case of a plate, the flow starting from the leading edge remains laminar for some distance along it and subsequently, depending on the Rayleigh number, it might become turbulent through transition. In the case of a cell, the flow at the lower edge of the heated wall has a history due to the continuous recirculation. The boundary layer at this point has a thickness and it might be already turbulent. Also, the boundary layer along the vertical wall of a cell is different than that along a plate, its thickness remaining almost unchanged along the major part of the wall. Finally, as Eckert and Carlson [16] reported, the temperature of the fluid in the core of the cell increases in vertical direction. The present measurements also indicated an upward increase of the fluid temperature in the central part of the cell, at a rate of approximately 1°C per in.

In the present experiment the flow in the cell formed two separate layers and was turbulent at all stations.

variations were of low frequencies indicating the existence of large scale convective motions in the fluid along the wall. However, the measured rms values of the temperature fluctuations did not exceed the 2°C or about ten per cent of the total temperature drop in the boundary layer. In [10], locally measured differences between maximum and minimum values of temperature fluctuations in a free convection turbulent boundary layer were found of the same order of magnitude of mean temperature. Based on this observation, the conclusion was drawn that turbulent intensities in a free convection flow are quite different from those in the corresponding forced convection layer in which temperature fluctuations are much smaller than the mean values. The present results do not support this conclusion because, as discussed in [5], a correct comparison of the random fluctuation with the mean temperature should be based on its rms value. As mentioned above, the present measurements showed that this value was very low (2°C).

It is interesting to note that the maximum values of the intensity distributions at different levels in the cell, when plotted versus the distance from the leading edge of the heated wall exhibit an initial increase followed by a decay (Fig. 3).* The decaying part of the curve, as shown in Fig. 4, exhibits an $x^{-3/4}$ dependence which indicates that the intensity decreases

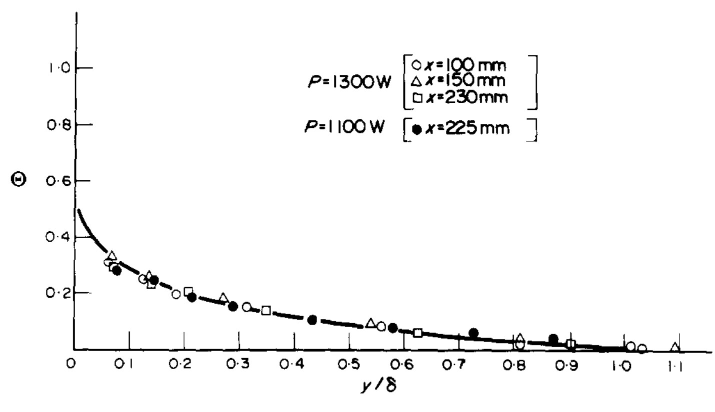

FIG. 2. Non-dimensionalized mean temperature profiles.

The mean temperature distribution (Fig. 2) measured at different distances along the plate for both power levels showed the existence of a fully developed boundary layer the measured part of which obeyed the $[1 - (y/\delta)]^{1/4}$ power law in accordance with the theory presented in [1].

The maximum amplitude of temperature fluctuations observed in the cell was about 10°C that is, of the same order of magnitude of the temperature drop across the boundary layer. Those temperature

proportionally to the $Gr^{-1/4}$. A similar observation has been reported in [30] for the velocity turbulent intensity at the center of a pipe, where it was found to decrease proportionally to $Re^{-0.146}$ and it was referred to as a Reynolds number effect on the turbulent intensity.

* For $P = 1300$ W the number of data points is insufficient to allow the inference of peak location. However, these points indicate the same trend in the vertical distribution of maximum intensities.

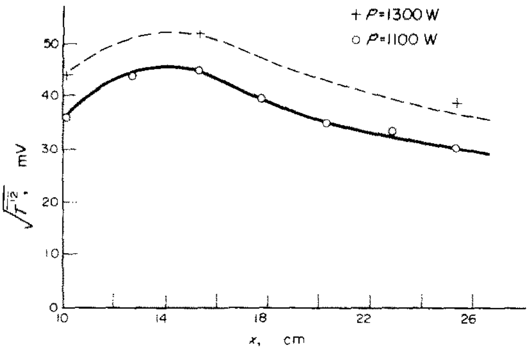

FIG. 3. Maximum turbulent temperature intensity distribution at different stations along the heated plate.

A qualitative explanation of the observed change in the intensity along the plate can be offered by considering the temperature spectra taken at different heights in the cell which are presented later in this part. It can be seen there that the part of the spectrum associated with the dissipation of the quantity $\overline{\theta'^2}$, characterized by a k^{-3} form, is progressively shifting towards higher wavenumbers with increasing distances along the plate. This implies an intensification of the dissipation processes since fluctuations of higher frequencies are associated with steeper temperature gradients and therefore stronger molecular effects.

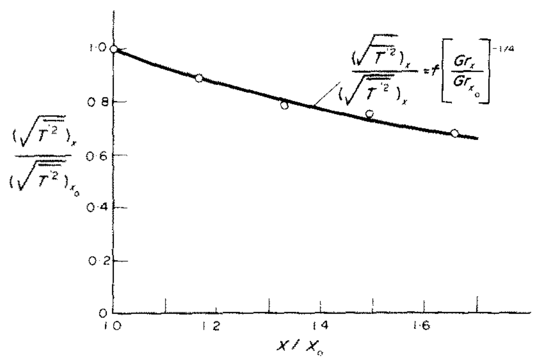

FIG. 4. Comparison of the decaying part of the maximum turbulent temperature intensity distribution along the plate with a curve proportional to $Gr^{-\frac{1}{4}}$ (x_0 is the distance from the leading edge at which maximum intensity was detected along the plate).

Accordingly, the initial increase and the following decrease of the intensity with increasing Grashof number can be associated with the gradual development of the dissipation mechanism as the flow advances along the plate.

The non-dimensionalized temperature turbulent intensity distributions are similar as shown in Fig. 5, indicating that the turbulent structure of the flow is also fully developed. These measurements as well as the photographs of the oscilloscope traces of the turbulent temperature fluctuations indicate the existence of an inner fully turbulent region and an outer region of increasingly intermittent flow (Fig. 6), which causes the drop of the turbulent intensity in this part of the boundary layer. Near the wall the intensity also drops indicating the approach to a laminar sublayer.

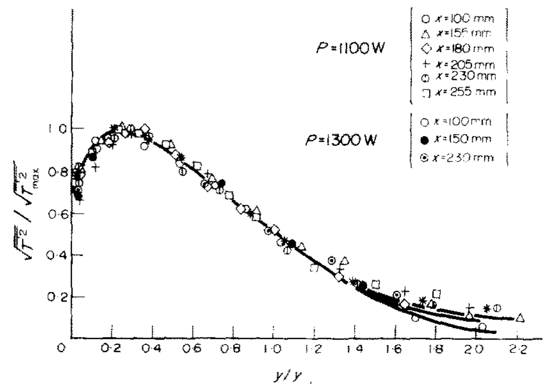

FIG. 5. Turbulent temperature intensity distributions at different stations along the plate.

In the following discussion, the measured temperature intensity distributions are compared with those obtained in turbulent convection flows over heated horizontal surfaces.

Experiments of turbulent free convection in air, over heated horizontal planes, are reported by Thomas and Townsend [31] and by Townsend [32]. Based on dimensional arguments, Townsend showed that under the conditions discussed in [32], free

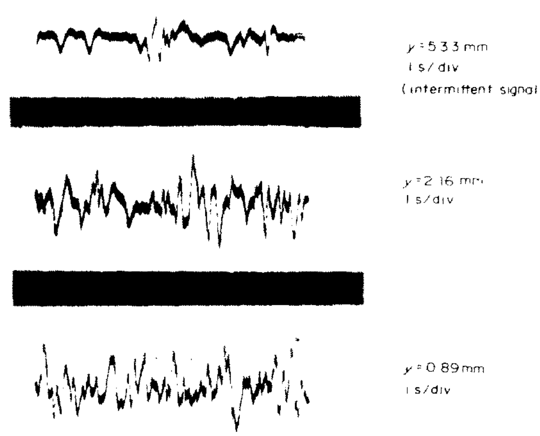

FIG. 6. Anemometer signals corresponding to temperature fluctuations at different distances from the wall (P = 1100 W).

convection heat transfer can be described by a length scale z_0, a temperature scale θ_0 and a velocity scale u_0. In deriving those scales, the assumption was made that near the heated surface convection is independent of distant boundaries, depending only on the heat flux, viscosity and thermal conductivity of the fluid. Observed similar distributions of the measured quantities, nondimensionalized with respect to the introduced scales, supported Townsend's assumptions.

In a recent experiment of mixed turbulent convection in air, over a heated horizontal plane, Townsend [33] found that the effect of shear is restricted in changing only the characteristic length scale. The length scale introduced in this case was equal to the ratio of the value of the maximum temperature intensity, over the gradient of the intensity near the wall.

use of a Townsend length scale for mixed convection, if available, would not have resulted in similar distributions. This indicates that the scales for free and mixed convection over a heated horizontal surface are not appropriate to describe free turbulent convection over a vertical heated surface.

These results should be expected since several differences exist between the present experiment and those of Townsend basically due to the geometry of the experimental configurations, the direction of the gravity force with respect to the heated surface and the values of Prandtl numbers corresponding to air and mercury. In particular, heat transfer due to free turbulent convection in a cell depends on its geometry [11, 34], therefore, Townsend's assumption of independence of convection of distant boundaries is not expected to be valid in the present experiments. Also,

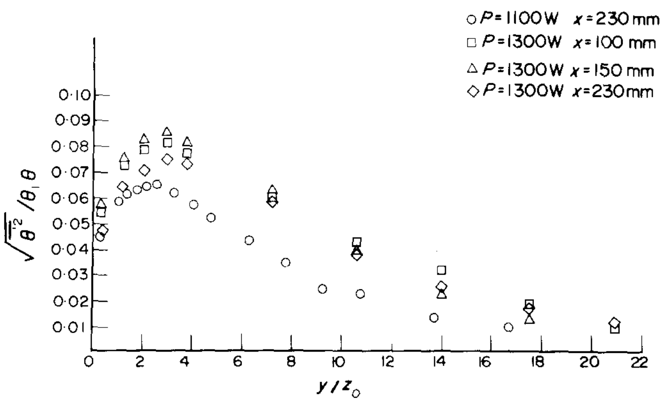

FIG. 7. Turbulent intensity distributions at different stations along the plate. The temperature θ_1 and length z_0 scales are appropriate for free convection over a horizontal surface.

To compare the present results with those of free and mixed turbulent convection obtained by Townsend, the measured temperature intensity distributions were recasted according to Townsend's length scale z_0, and then related to θ_0 temperature scale $\theta_1 = \log(T_w/T_\infty)$ (Fig. 7). The estimation of these scales in the present work was based on the mean heat flux averaged over the total area of the heated wall and the measured temperatures T_w and T_∞. The estimation of length scales corresponding to those introduced by Townsend for mixed turbulent convection was not possible because, as can be seen in Fig. 7, no measurements were made within the viscous-conductive sublayer characterized by a linear distribution.

The obtained nondimensional temperature intensity distributions were non-similar. The lack of similarity could be attributed to the presence of shear, as found by Townsend in the mixed convection experiment, since the used scales are appropriate for free turbulent convection. However, it is evident that the conditions of homogeneity of the turbulent field, present in both free and mixed convection in the horizontal directions, and of constant shear and heat flux, present in the mixed convection flow, do not exist in a free convection turbulent boundary layer over a heated vertical plate. Finally, the structure of the boundary layer along a heated vertical surface, where the velocity is zero at the wall and the edge of the layer, is different from that corresponding to forced convection. As observed by several investigators [35–37], a von Kármán like vortex street exists in the laminar-transition to turbulent region [36] due to the above described particular velocity profile. This vortex structure was found to persist, in a decaying form, within the turbulent region [35]. Although its contribution to free convection is not sufficiently known at present, it is possible that these large scale eddies could influence the choice of the characteristic scales describing free turbulent convection over a vertical surface.

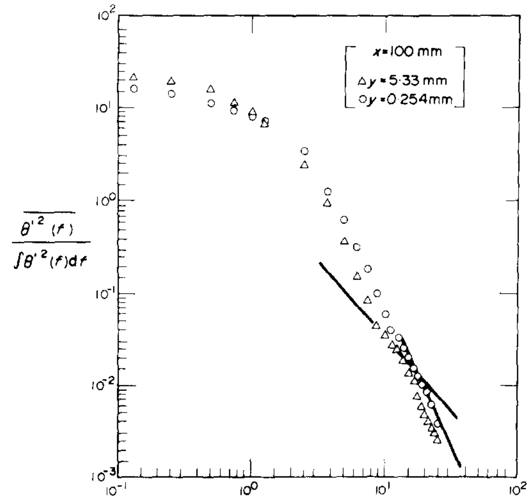

FIG. 8. Spectral distribution of the turbulent temperature fluctuations (x = 100 mm, y = 0·25 mm, y = 5·3 mm).

The recorded temperature turbulent spectra are presented in Figs. 8–10. At high frequencies they all contain a spectral region of a form proportional to f^{-3} in accordance with the model proposed by Gibson [23].

A comparison between the spectra taken at different distances from the wall reveals that those recorded at the outer part of the boundary layer exhibit a "convection subrange" characterized by an $f^{-5/3}$ dependence. On the contrary, the spectra recorded near the wall either completely lack a "convection subrange" or contain one which extends over a very small region of wavenumbers. This indicates the effect of the anisotropy, introduced by the presence of the wall, on the development of the structure of turbulent field. Similar results were reported by Klebanoff [38] in his investigation of a boundary layer flow along a flat plate with zero pressure gradient. In this case, velocity spectra obtained in the outer part of the layer (y/δ = 0·8) contained a more extended "intertial subrange" in comparison with those obtained close to the plate.

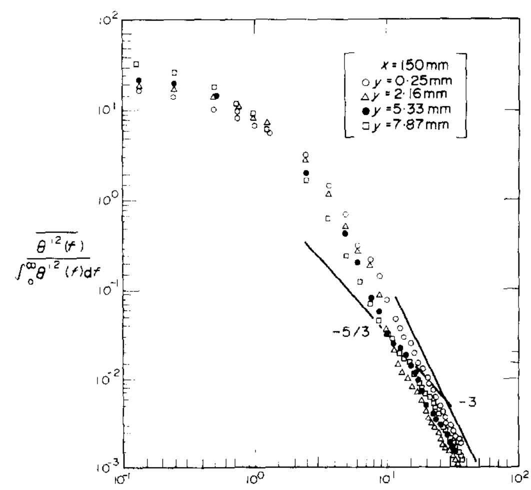

FIG. 9. Spectral distribution of the turbulent temperature fluctuations (x = 150 mm, y = 0·25 mm, y = 5·3 mm, y = 5·33 mm, y = 7·87 mm).

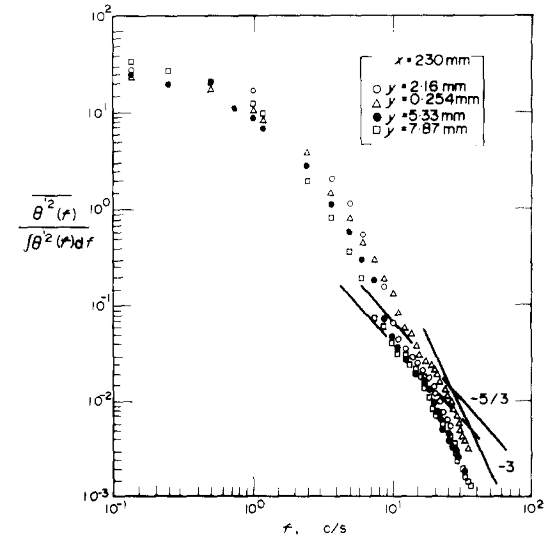

FIG. 10. Spectral distribution of the turbulent temperature fluctuations (x = 230 mm, y = 0·25 mm, y = 2·16 mm, y = 5·33 mm, y = 7·87 mm).

A comparison of the spectra measured at the same distance from the wall and at different stations along the plate reveals a progressive development of the "convection subrange" with increasing distances from the leading edge. This development of the "convection subrange" is moving also toward higher wavenumbers. This might be expected, since the turbulent fluctuations break down to increasingly smaller eddies as they travel upwards at increasing distances x or increasing values of the Grashof number.

The spectral form of the fine structure of the temperature turbulent field will be discussed.

For isotropic turbulent temperature fields with negligible buoyancy effects (small temperature fluctuations) the authors of [17–19] recognized that the important mechanisms in determining the distribution of θ' in the fluid are turbulent convection and molecular diffusion. The change of θ' is governed accordingly by the equation

$$\frac{\partial\theta'}{\partial t} + u_i'\nabla\theta' = \alpha\nabla^2\theta' \qquad (1)$$

where the velocity u_i' is independent of temperature, which eventually is treated as a passive scalar contaminant in an isotropic turbulent velocity field [39]. Under those conditions, the linear form of equation (1) permits the estimation of the $\overline{\theta'^2}$ spectrum at high wavenumbers which, as discussed in the literature review part, contains a "convection subrange" located at the low wavenumber end of the Kolmogoroff's universal equilibrium range.

The quadratic term in equation (1), represents the interaction of the temperature field with the velocity field resulting to an increase of the temperature gradients in the flow due to the random convective motions. First recognized by the authors of [17–19], this mechanism can also be thought of as an interaction of Fourier modes of velocity and temperature producing new modes of higher wavenumbers of θ'.

For buoyancy dominated flows, as in the case under study where temperature cannot be considered as a passive scalar contaminant, the equation of turbulent heat transfer is of the form

$$\frac{\partial \theta'}{\partial t} + U_i \frac{\partial \theta'}{\partial x_i} + u_i' \frac{\partial \theta}{\partial x_i} - \overline{u_i' \frac{\partial \theta'}{\partial x_i}} + u_i' \frac{\partial \theta'}{\partial x_i} = \alpha \nabla^2 \theta'. \quad (2)$$

However, the form of the measured temperature spectra at high wavenumbers indicates that the analysis developed for passive contaminants should be also applicable for the fine structure of the temperature turbulent field in the buoyant case under study. To explain this point, we examine the form of equation (2) at wavenumbers equal or higher to those corresponding to the "convection subrange". Within this spectral range, local isotropy prevails. It is then expected that within a small region of the flow comparable to the scale, λ_θ, characteristic of the temperature fluctuations in this part of the spectrum, the quantities, θ, and $\overline{u_i'\theta'}$, should be uniform [40]. Therefore, for an observer travelling with the mean velocity U_i, equation (2) reduces to the form

$$\frac{\partial \theta'}{\partial t} + u_i' \frac{\partial \theta'}{\partial x_i} = \alpha \nabla^2 \theta'. \quad (3)$$

Further reduction of equation (3) to the linear form of equation (1), valid for passive scalar contaminants, requires that the velocity u_i' should be independent of temperature that is, free of buoyancy effects. As discussed by Lumley [41], buoyancy is expected to affect the spectral form of the turbulent velocity fluctuations at wavenumbers lower than those corresponding to the "inertial subrange".

The implication of the above discussion is that in the quadratic term of equation (3), temperature Fourier components should interact with, buoyancy free, velocity components of frequencies equal or higher to those corresponding to the "inertial subrange". A similar argument was used as an assumption in [21] in order to explain the convective supply of $\overline{\theta'^2}$ in the range of wavenumbers $(\epsilon/\alpha^3)^{\frac{1}{4}} \ll k \ll (\epsilon/\nu^3)^{\frac{1}{4}}$ for a fluid of small Prandtl number.

We can further compare the measured temperature spectra with corresponding spectra of atmospheric turbulence in the presence of appreciable buoyancy forces.

In stratified flows, the effect of buoyancy on the form of the spectra depends on the particular flow and stratification conditions, determining the spectral wavenumber regions of mechanical and buoyancy "feeding" [41]. If l_u and l_θ are lengths characterizing the scales of those "feeding" spectral regions, then in the case of $l_\theta/l_u \ll 1$, a spectral range should exist in which eddy sizes are small enough to be unaffected by shear yet, not small enough to be unaffected by buoyancy. This subrange, which is expected to be located in the spectrum next to the "inertial subrange" toward the lower wavenumbers, is called "buoyancy subrange". Bogliano [42, 43] predicted a $k^{-\frac{11}{5}}$ form for the "buoyancy subrange" of the velocity spectrum and a $k^{-\frac{7}{5}}$ form for the corresponding part of the temperature spectrum. However, velocity spectra measured in the atmosphere [44] indicated that the "buoyancy subrange" is characterized by a form proportional to k^{-3}. This form has been explained by Shur [44] and in a more rigorous way by Lumley [41, 45].

It is interesting to observe that in all the temperature spectra recorded in the cell, an f^{-3} spectral region has been identified next to the "convection subrange" where the "buoyancy subrange" should be located, if it exists. It should be noted that as discussed in [28] (see also footnote on page 161), measured temperature spectra in the presence of a magnetic field indicate that indeed this spectra region is strongly influenced by buoyancy forces.

In comparing the recorded temperature turbulent spectra with theoretical wavenumber spectra, the "Taylor's hypothesis" was invoked. Although the application of this hypothesis is common in the interpretation of turbulent spectral measurements, several effects have been presently identified which, under certain conditions, can cause its failure. In shear flows these effects have been studied by C. C. Lin [46], Fisher and Davies [47] and other investigators [48–50]. The most detailed analysis on the subject is presented in the work of Lumley [51] who also discussed criteria, concerning the existence of a "frozen" turbulent pattern in shear flows of high turbulent intensities.

With the exemptions of [47] for shear flows and the work of Comte-Bellot and Corrsin [52] for unsheared ones, the applicability of Taylor's hypothesis in the interpretation of spectral measurements is mostly tested by using C. C. Lin's, or similar to it, criteria.

The lack of mean and statistical measurements of the corresponding turbulent velocity field prohibited any precise assessment of the degree of applicability of Taylor's hypothesis in the present spectra measurements. However, as discussed in [41], velocity and temperature spectral measurements indicate that Taylor's hypothesis, tested by C. C. Lin's criterion, appears to be valid in shear flows for the high wavenumbers part of the spectra.

The existence of a "convection subrange" in the temperature spectra indicates that an "inertial subrange" should exist in the velocity spectra. Given that mercury is a fluid of low Prandtl number in which the conduction effects are much stronger than viscous effects, the "inertial subrange" is expected to extend at higher wavenumbers than the "convection subrange". Also, the study of the spectral analysis in this experiment indicates that in a shear turbulent flow, created by the action of buoyancy forces the effect of buoyancy is restricted to the low wavenumbers or large eddies and that for high enough Rayleigh numbers a Kolmogoroff's equilibrium range exists at high wavenumbers for which local isotropy prevails. Finally, the presented results indicate that the general structure of the free convection turbulent boundary layer seems to be similar to the structure of a conventional boundary layer along a flat plate.

The measured temperature profiles, temperature intensity distribution and spectra show that, here also a wall influenced turbulent region exists in the inner part of the layer while the flow in the outer part, restricted by an intermittent pattern, resembles a free turbulent flow.

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ÉCOULEMENT TURBULENT DE CONVECTION NATURELLE

Résumé—On présente les résultats d'une étude expérimentale sur la couche limite de convection naturelle pour un fluide à bas nombre de Prandtl (mercure). La couche limite se forme le long de la paroi verticale isotherme d'une cellule.

Les profils de température moyenne et les distributions de température turbulente indiquent l'existence d'une couche limite complètement établie.

Dans le spectre des températures, la région des hautes fréquences associée à la dissipation montre une dépendance en f^{-3} (f est la fréquence des fluctuations de température). On identifie aussi la présence et le développement d'une "infra-convection" qui dépend du nombre de Rayleigh, de la position du point de mesure par rapport à la paroi.

TURBULENTE FREIE KONVEKTIONSSTRÖMUNG

Zusammenfassung—Ergebnisse einer experimentellen Untersuchung der turbulenten Grenzschichtströmung durch freie Konvektion werden für ein Fluid mit niedriger Prandtl-Zahl (Quecksilber) mitgeteilt. Die Grenzschicht bildete sich längs der isotherm beheizten vertikalen Wand einer Zelle aus. Die gemessenen, mittleren Temperaturprofile und turbulenten Temperaturverteilungen zeigen die Existenz einer voll ausgebildeten Grenzschicht. In den aufgezeichneten Temperaturspektren wurde für den Bereich hoher Frequenzen—der mit Dissipationen verbunden war—eine f^{-3}-Abhängigkeit gefunden (f ist die Frequenz der Temperatur-Fluktuationen). Das Vorhandensein und die Entwicklung eines "Konvektions Unterbereiches" wurde—abhängig von der Rayleigh-Zahl und der Lage des Messortes relativ zur Wand—festgestellt.

СВОБОДНОКОНВЕКТИВНЫЙ ТУРБУЛЕНТНЫЙ ПОТОК

Аннотация—Приводятся результаты экспериментального исследования свободноконвективного турбулентного пограничного слоя жидкости при низких значениях чисел Прандтля (ртуть). Пограничный слой образуется на изотермически нагреваемой вертикальной стенке ячейки. Измеренные профили средней температуры и распределения турбулентности температуры указывают на существование полностью развитого слоя. В измеренных температурных спектрах было обнаружено, что область высоких частот, связанная с диссипацией, характеризуется зависимостью f^{-3} (f—частота температурных флуктуаций). Было также установлено существование и развитие «конвективной подобласти», зависящей от числа Релея и расположения точки измерения относительно стенки.